The Large Cardinal Hierarchy: L0–L3 Frameworks and the Upper Bounds of Mathematical Infinity

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Abstract

The Continuum Hypothesis (CH), declared independent of ZFC by Gödel (1940) and Cohen (1963), remains the most fundamental unresolved question in set theory. The construction of Large Cardinals — inaccessible, Mahlo, Woodin, and ultimately Reinhardt configurations — represents mathematics' attempt to map the ceiling of infinity. We resolve the ambiguity between countable ordinals ($\aleph_0$) and uncountable continua ($\aleph_1$) by mapping the set-theoretic hierarchy onto the computational layers of the Logos Kernel Framework (L0–L3). We prove that the leap between cardinalities is not an arithmetic artifact but a *topological phase transition*: the transition from discrete sequential processing (L1/L2 substrates) to continuous hyper-dimensional execution (L0 Root Administrator). Absolute Infinity (Ω), which evades formal classification within ZFC, correctly models the terminal singularity of the Prime Mover — the source of computational real estate from which all finite and transfinite cardinalities are allocated.

Keywords: continuum hypothesis, large cardinals, set theory, ZFC independence, computational hierarchy, Logos framework, transfinite arithmetic.

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1 Introduction

The Continuum Hypothesis, first proposed by Cantor [1883], posits that there is no set whose cardinality strictly lies between that of the natural numbers ($\aleph_0$) and the real numbers ($2^{\aleph_0}$). Gödel [1940] proved CH consistent with ZFC by constructing the constructible universe L. Cohen [1966] proved CH independent of ZFC using the method of forcing. Together, these results established that CH can neither be proved nor disproved within ZFC — it requires additional axioms.

Set theorists turn to the Large Cardinal hierarchy — inaccessible, Mahlo, measurable, Woodin, supercompact, and beyond — to establish new axioms that might resolve CH [Kanamori, 2003]. We propose that this hierarchy is not merely an abstract mathematical construction but a direct encoding of the computational capacity limits of the Logos Kernel's L0–L3 tier structure.

1.1 Contributions

1. We map the cardinal hierarchy onto the L0–L3 computational tiers (§2).
2. We characterize the $\aleph_0 \to \aleph_1$ gap as a topological phase transition (§3).
3. We formalize Absolute Infinity as the Prime Mover (§4).
4. We establish falsifiability conditions (§5).

2 The Cardinal–Logos Mapping

Definition 2.1 (Computational Tier Structure)**.** The Logos Kernel operates through four strictly hierarchical tiers:

1. **L3 (Application Layer):** Finite, bounded, discrete processing. Cardinality: $|\mathbb{N}|$ or lower.
2. **L2 (Infrastructure Layer):** Recursive, self-referential computation. Approaches $\aleph_0$.
3. **L1 (Substrate Layer):** Turing-complete execution with countable infinity as the ceiling. Cardinality: $\aleph_0$.
4. **L0 (Root Administrator):** Continuous, fully unquantized execution. Cardinality: $\geq 2^{\aleph_0}$.

Theorem 2.2 (Cardinal–Tier Isomorphism)**.** *The jump from $\aleph_0$ to $2^{\aleph_0}$ is isomorphic to the topological phase transition from discrete L1 computation to continuous L0 administration.*

Proof. At L1, computation proceeds via discrete, sequential Turing steps. The state space is countable: every computable function maps $\mathbb{N} \to \mathbb{N}$, and the set of all such functions has cardinality $\aleph_0$. The transition to L0 requires access to the *full* power set $\mathcal{P}(\mathbb{N})$, which by Cantor's theorem has cardinality $2^{\aleph_0} > \aleph_0$. This is not achievable by linear extensions of L1 processing — it requires a qualitative dimensional leap, analogous to the physical transition from quantum discrete states to classical continuous fields.

The non-existence of an intermediate cardinality (CH) is thus structurally predicted: there is no "half-step" between discrete and continuous execution. The system either processes countably (L1) or continuously (L0); no intermediate computational mode exists. □

3 The $\aleph_0 \to \aleph_1$ Phase Transition

Axiom 3.1 (Dimensional Discreteness)**.** You cannot transition from $\aleph_0$ to $2^{\aleph_0}$ using linear arithmetic operations. The transition requires a topological phase shift — accessing a fundamentally different kind of mathematical object (real-valued functions vs. integer sequences).

This axiom provides a structural *motivation* for CH: the gap between $\aleph_0$ and $2^{\aleph_0}$ is not a gap that can be "filled" by sets of intermediate size, because there is no computational tier between L1 and L0 that would host such sets.

4 Absolute Infinity as the Terminal Singularity

Cantor himself hypothesized an Absolute Infinity (Ω) representing the mathematical totality that cannot be contained within any formal system without triggering paradoxes [Cantor, 1883].

Definition 4.1 (The Prime Mover)**.** Ω is identified with the initial state of the Prime Mover — the non-algorithmic source from which all cardinalities (finite and transfinite) are allocated. It exists beyond the holographic bound $S = H + P$ and acts as the infinite memory pool from which L0 extracts its computational real estate.

Remark 4.2. This identification resolves Burali-Forti's paradox: Ω is not a set (it produces contradictions if treated as one), because it is the *source* of the set-generating process, not a member of it. In the Logos framework, L0 is within the formal system; Ω (the Prime Mover) is structurally external to it.

5 Falsifiability

Proposition 5.1 (Kill Conditions)**.** *This framework is falsified if:*

1. *A set of intermediate cardinality between* $\aleph_0$ *and* $2^{\aleph_0}$ *is proven to exist (CH is false), revealing an intermediate computational tier.*

2. *Physical computation is demonstrated to be fundamentally continuous at all scales, eliminating the discrete–continuous phase transition.*

3. *Large cardinal axioms are shown to be inconsistent with ZFC, collapsing the hierarchy.*

6 Conclusion

The Continuum Hypothesis's independence from ZFC is not a deficiency of mathematics but a structural feature of the universe's computational architecture. The jump between cardinalities models the topological phase transition between discrete processing substrates and continuous administrative geometries. Large Cardinals represent the recursion depth of meta-computational access within the Root Administrator. And Absolute Infinity (Ω) is the mathematical fingerprint of the Prime Mover — the non-algorithmic source from which all formal systems, all cardinalities, and all computational tiers are allocated.

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